

**Modeling Fundamentals:
Concepts of Models and Systems
Concepts of Modeling
Classifications of Models**

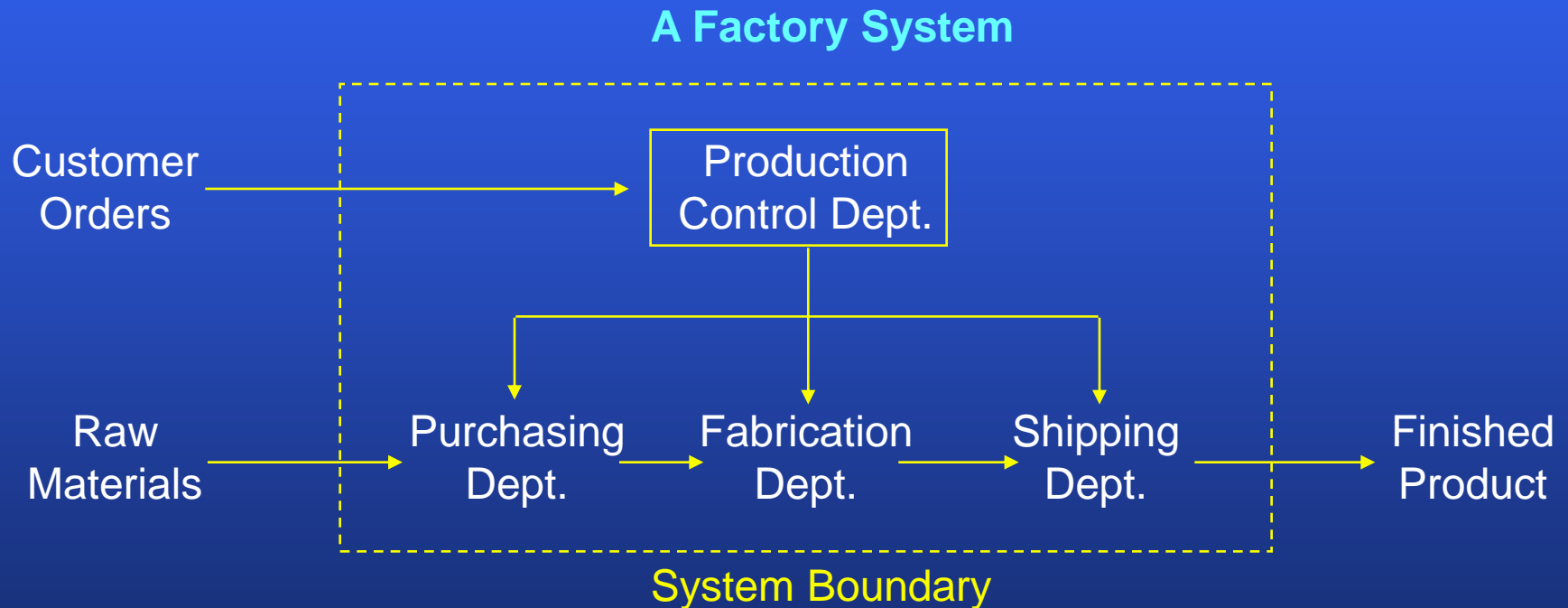
OSMAN BALCI
Professor

Department of Computer Science
Virginia Polytechnic Institute and State University (Virginia Tech)
Blacksburg, VA 24061, USA

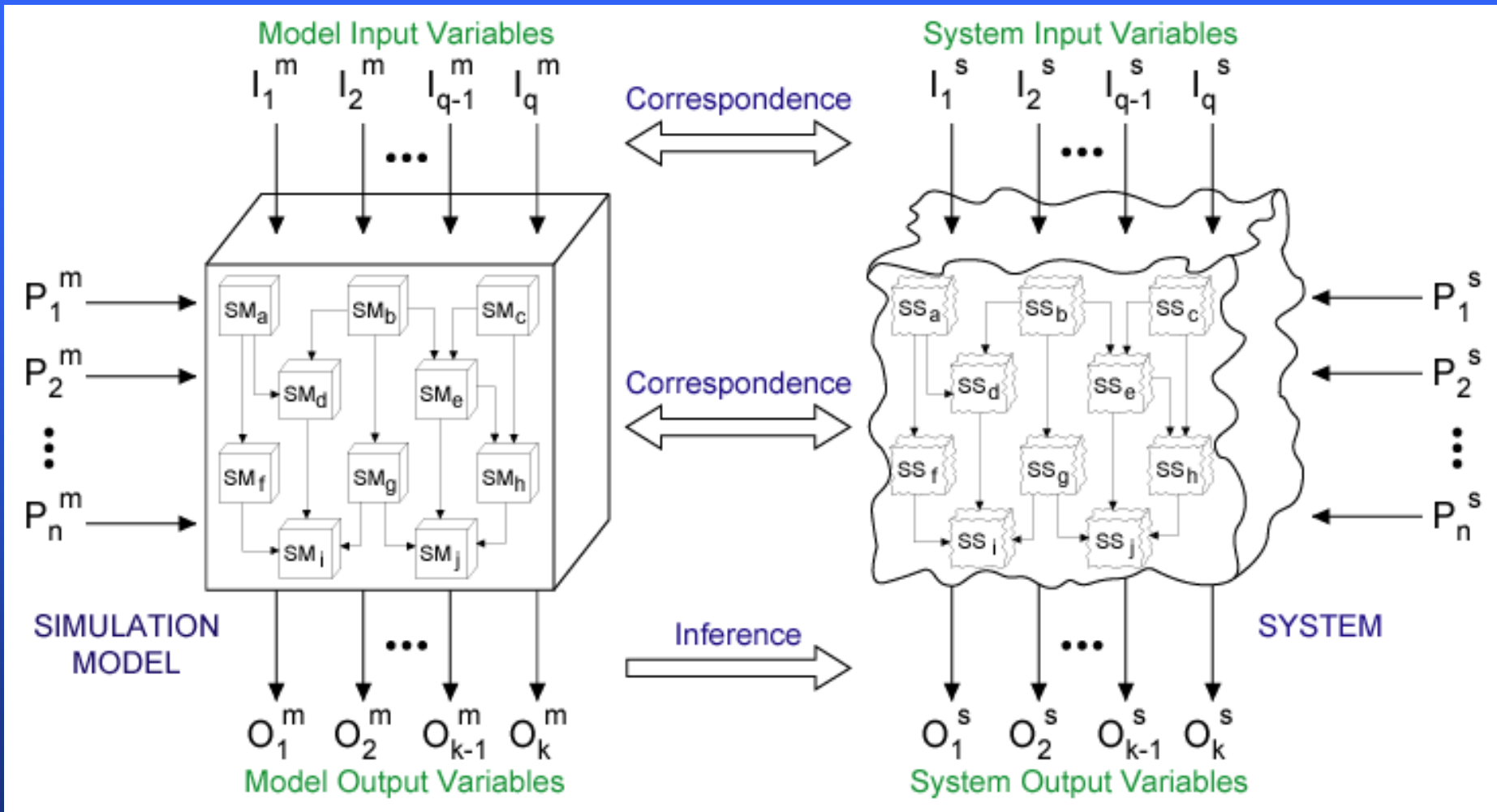
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The Concepts of a System

System: is any collection of interacting elements that operate to achieve some goal.



The Concepts of Modeling



I	→ Input	m	→ model	SM	→ Sub Model
P	→ Parameter	s	→ system	SS	→ Sub System
O	→ Output	q, n, k	→ integer number		

Definitions

Input Variable (I)

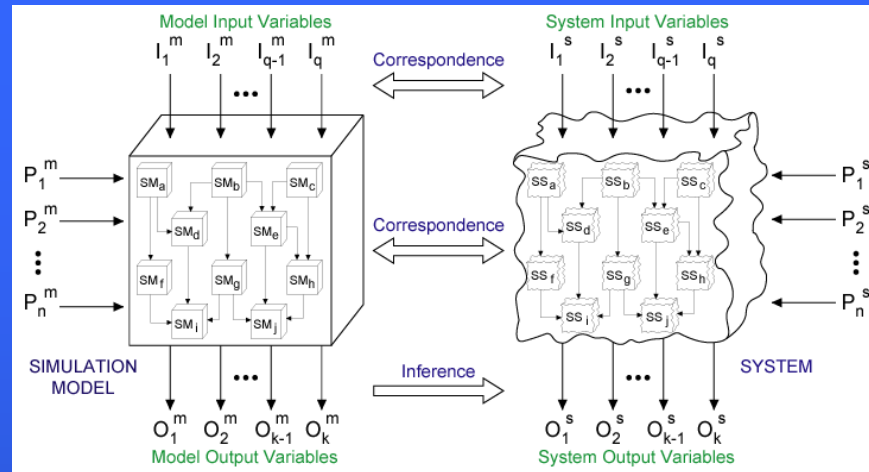
- represents a changing value used to characterize what physically or logically goes into system / model.

Examples for a Traffic Intersection System:

- ❖ I_1 = Arrivals of vehicles to lane 1
- ❖ I_2 = Arrivals of vehicles to lane 2
- ❖ I_3 = Arrivals of vehicles to lane 3
- ⋮
- ❖ I_n = Arrivals of vehicles to lane N

Parameter (P)

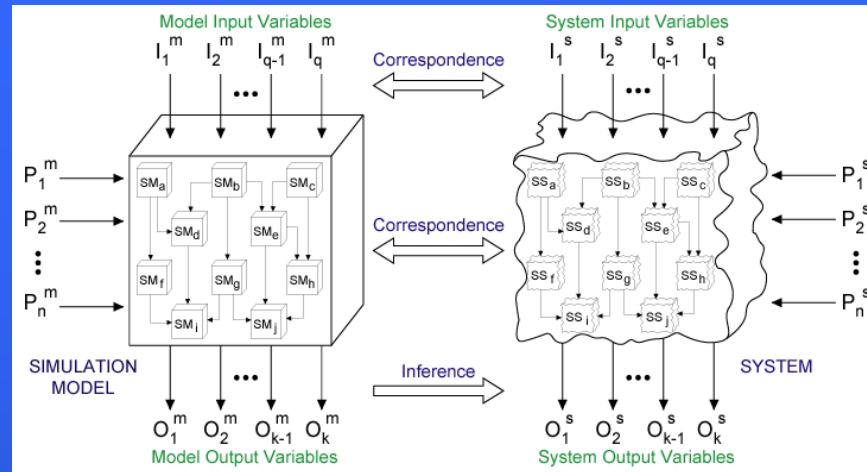
- is a constant value influencing system / model behavior.
- Examples for a Traffic Intersection System:
 - ❖ P_1 = Duration of red light
 - ❖ P_2 = Duration of green light



Definitions

■ Output Variable (O)

- represents a behavioral / operational characteristic of interest of the system or model.
- Also called:
 - ❖ Measure of Effectiveness (MOE)
 - ❖ Performance Measure
 - ❖ Response Variable
 - ❖ Technical Performance Measure (TPM)
- Examples for a Traffic Intersection System:
 - ❖ O_1 = Average vehicle waiting time in lane 1
 - ❖ O_2 = Average vehicle waiting time in lane 2
 - ❖ O_3 = Average vehicle waiting time in lane 3
 - ⋮
 - ❖ O_n = Average vehicle waiting time in lane N



Types of Problems

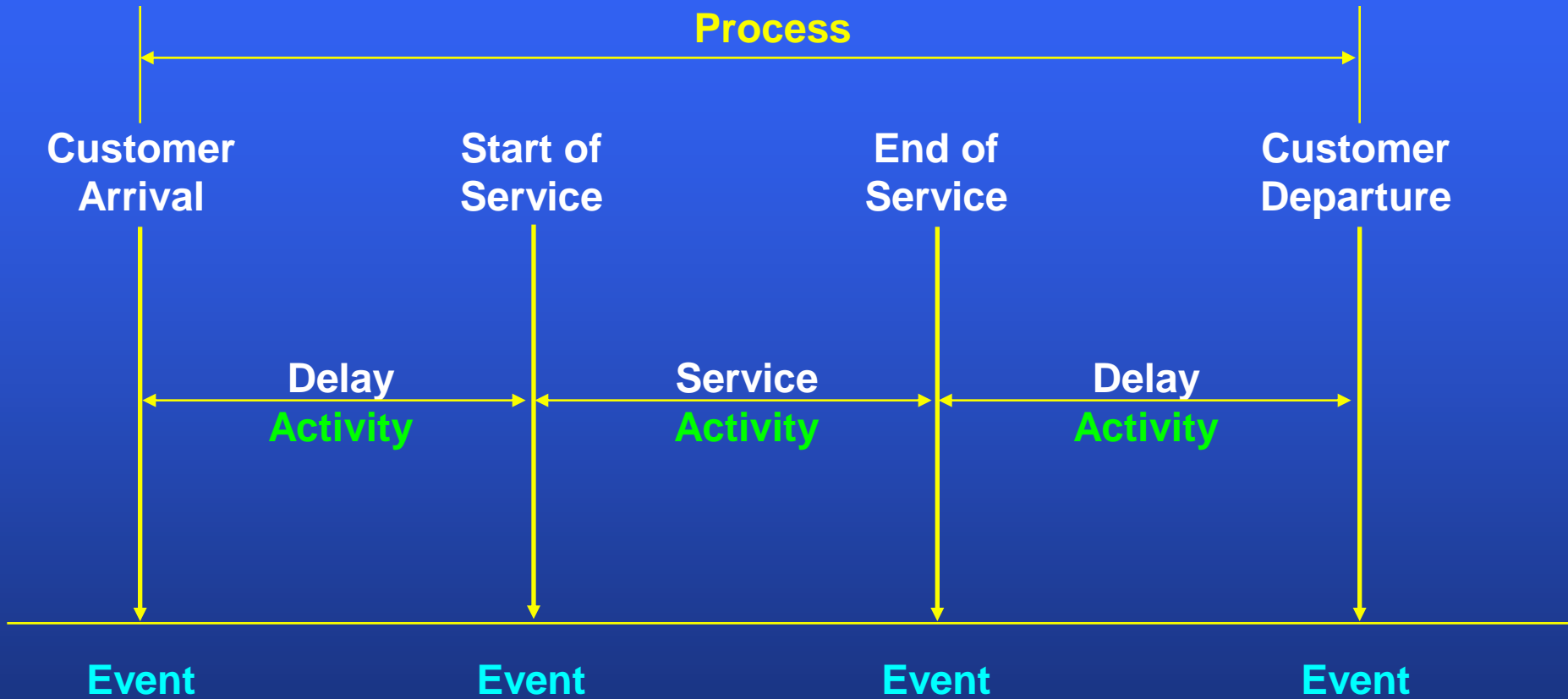
Problem Type	Given	Determine
Analysis (Direct)	Input, System	Output
Synthesis (Design Identification)	Input, Output	System
Instrumentation (Control)	System, Output	Input

System Description

A system can be described in terms of objects (entities), attributes, events, activities, and processes.

- An **Object** (or **Entity**) denotes an element of interest in the system.
- An **Attribute** denotes a property or a characteristic of an object or describes an aspect of an object.
- An **Event** is anything that occurs **at an instant of time t** and changes the state of the system.
- An **Activity** is an action that occurs **over an interval of time**. An activity starts with an event and ends with another event.
- A **Process** is the succession of states of an object over a span which is the contiguous succession of one or more intervals.

Event, Activity, Process



State

- The **State of an Object** is the enumeration of all attribute values of that object at a particular instant of time.
- The **State of a System** is the exhaustive enumeration of all attribute values (of all objects) at a particular instant of time.

Activity Types

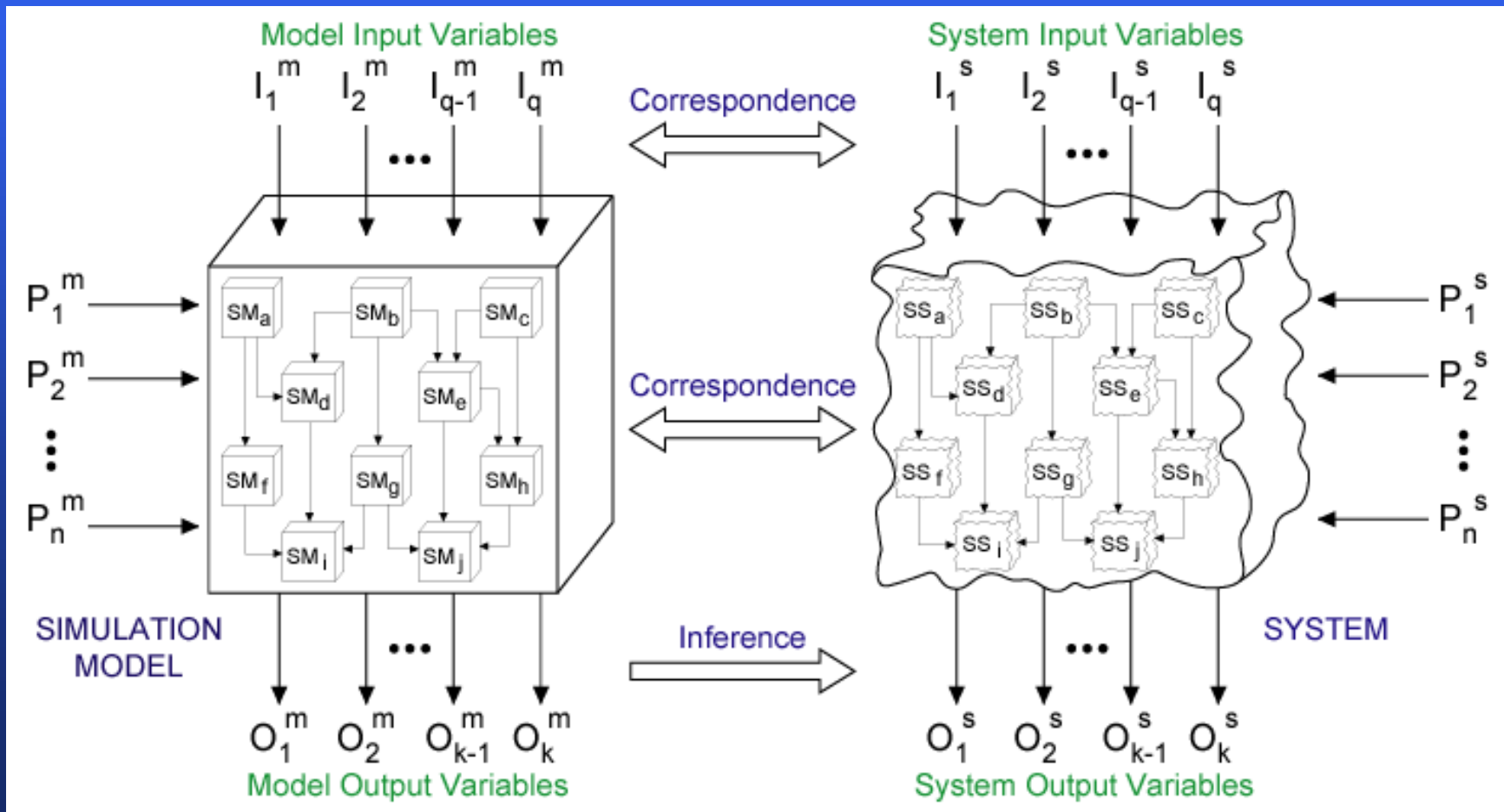
- **Endogenous Activity:** is produced within the system resulting from internal causes.
- **Exogenous Activity:** is produced within the system environment and affects the system. Exogenous activities are inputs to a system.
- **Deterministic Activity:** is an activity the outcome of which can be described completely in terms of its input.
- **Stochastic Activity:** is an activity the outcome of which varies randomly.

System Types

- **Open System:** is a system which has exogenous activities.
- **Closed System:** is a system which has no inputs (exogenous activities) and no outputs.
- **Continuous System:** is a system in which the changes are predominantly smooth such as a missile system.
- **Discrete System:** is a system in which the changes are predominantly discontinuous such as a factory system.
- **Adaptive System:** is a system which reacts to changes in its environment.
- **Nonadaptive System:** is a system which does not react to changes in its environment.

The Concepts of Models and Modeling

- A **Model** is a **representation** and **abstraction** of anything such as a real system, a proposed system, a futuristic system design, an entity, a phenomenon, or an idea.



Art of Modeling

- **Modeling** is an artful balancing of opposites;



Art of Modeling

- on the one hand, a model should not contain unnecessary details and become needlessly complex and difficult to analyze,



Art of Modeling

- on the other hand, it should not exclude the essential details of what it represents.



Art of Modeling: What to include? What not to include?

Example: Modeling of Coin Tossing

Tail



Head

- How should we model Coin Tossing?
- In our model representation, shall we **include** or **exclude**
 - Size? (penny, dime, nickel or quarter)
 - Coin composition? (copper, nickel, zinc)
 - Flipping it in the air or spinning it on its edge on a table?
 - Surface of landing area if flipping it in the air? Concrete, muddy land, or wood landing surface?
 - Possibility that the coin may land on its edge? Should the model output be Head, Tail, or Undecided?
 - Height and speed of human-generated flip?
 - The angle of catch if catching the flipped coin?

Art of Modeling: What to include? What not to include?

Example: Modeling of a Traffic Intersection?

[Click here to play video](#)



← Physical Boundary

Logical Boundary?

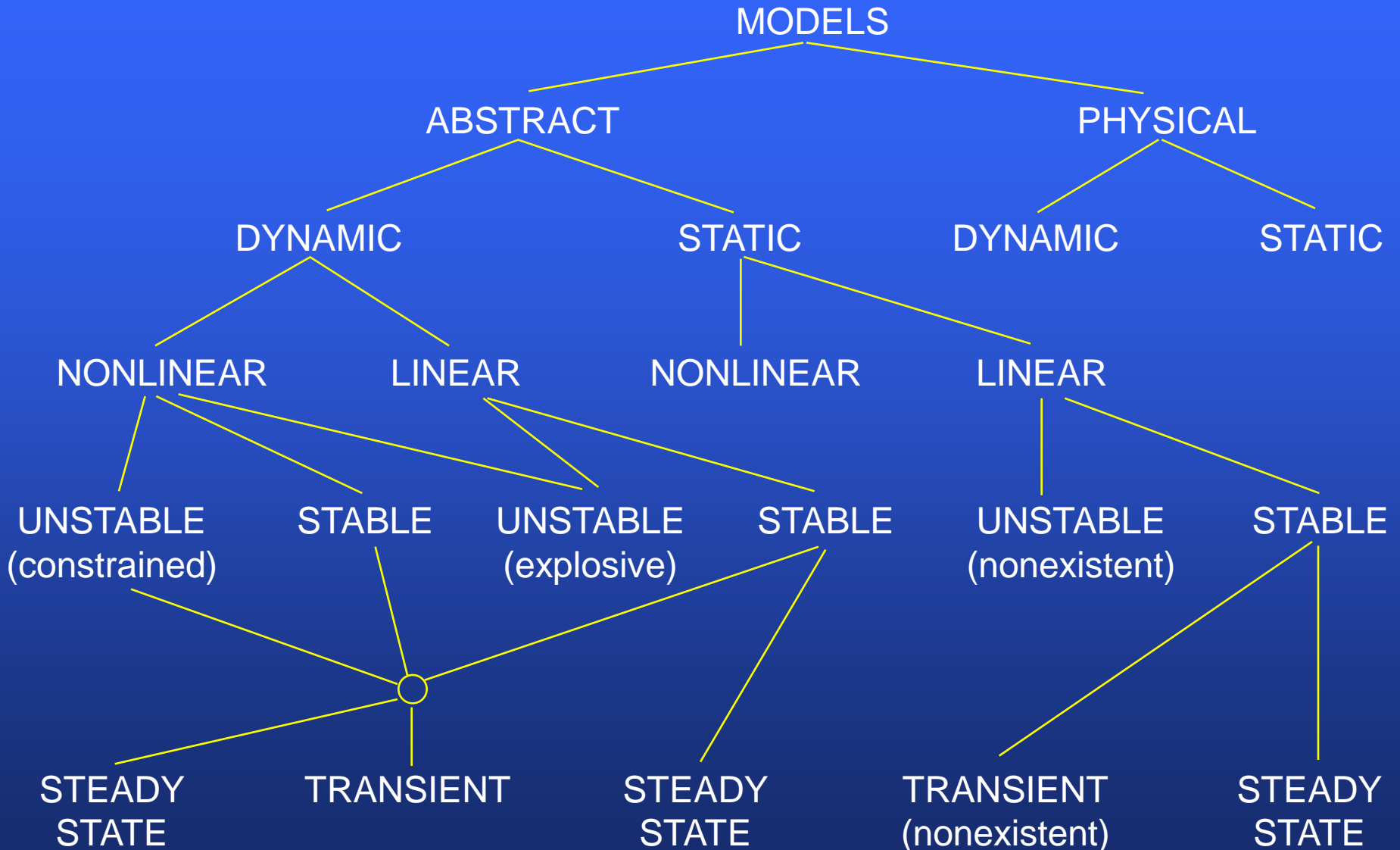
Shall we **include** or **exclude**

- Period of operation? Morning rush hour or the whole day?
- Pedestrian crossings?
- Right turn on red?
- Traffic light Yellow?
- Bicycles?
- Emergency vehicles?
- Accidents?

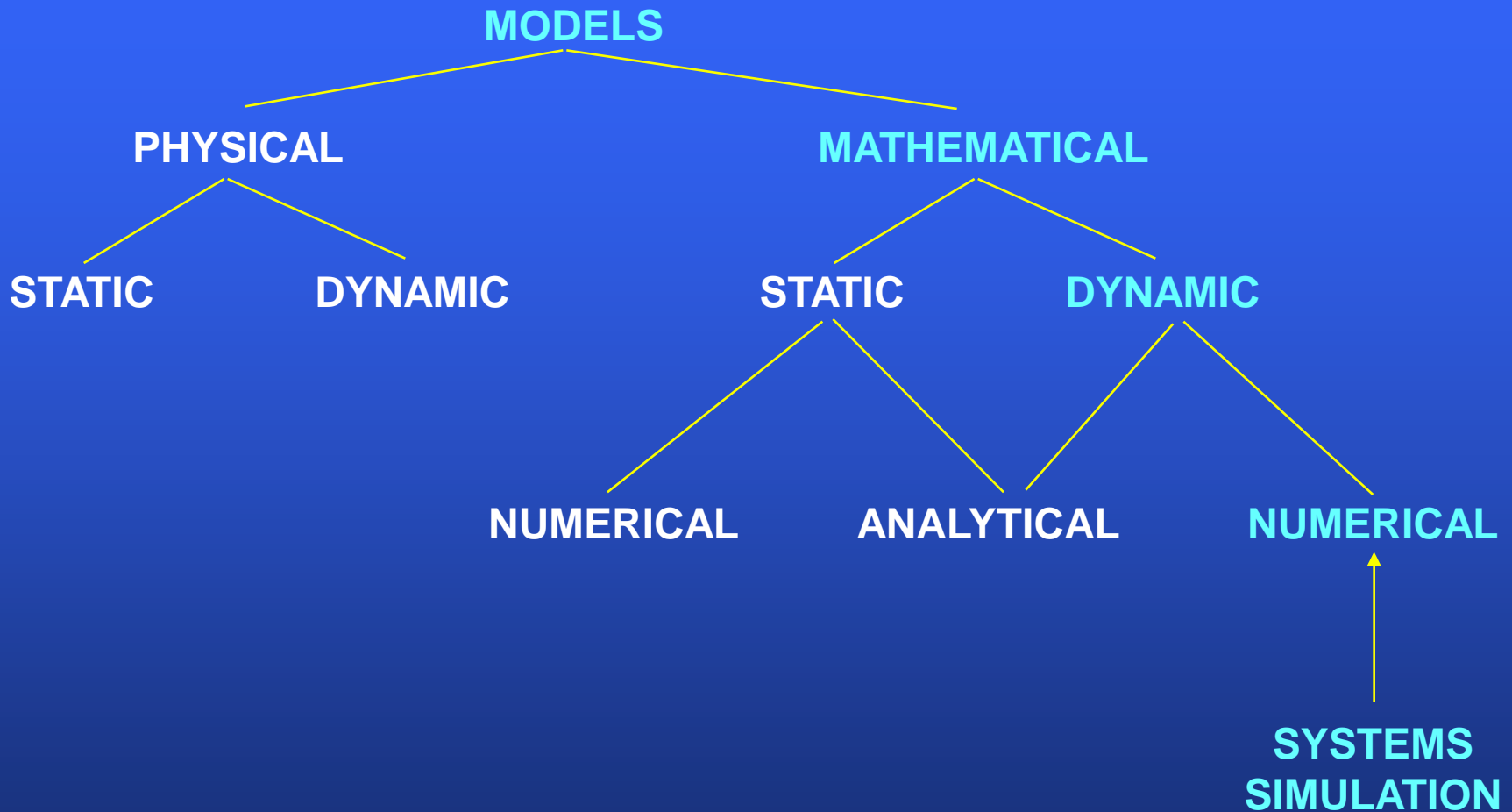
Fundamentals of Modeling

- Successfully accomplishing the artful balancing of opposites requires **significant experience and knowledge** not only in M&S, but also in the problem domain.
- In modeling studies, it is well to remember the dictum “**nobody solves the problem; rather, everybody solves the model that he or she has constructed of the problem.**” Hence, if the model does not represent the problem with sufficient accuracy, the modeling exercise turns itself into
 - ❖ “**junk-input junk-output**” or “**garbage-in garbage-out**”.
- **We never build a perfect model.** A model is built for a specific purpose and its accuracy is judged with respect to that purpose.

Classifications of Models



Classifications of Models



- **Static** implies that the model does not represent time.
- **Dynamic** implies that the model represents time.

Classifications of Models

ABSTRACT (MATHEMATICAL) OR PHYSICAL MODELS

- **Abstract (Mathematical) Model:** is the one in which symbols and logic constitute the model. The symbolism used can be a language or a mathematical notation.
 - A verbal or written description in English is an abstract model.
 - A mathematical model is described in the “language” of mathematical symbols and is an abstract model.
 - **A simulation model is built in terms of logic and mathematical equations and is an abstract model.**
- **Physical Model:** is the one which is usually a physical replica, often on a reduced scale, of the system it represents. A physical model “looks like” the object it represents and is also called an **Iconic Model**.
 - A model of an airplane (scaled down), a model of the atom (scaled up), a map, a globe, a model car are examples of physical (iconic) models.

Classifications of Models

STATIC OR DYNAMIC (ABSTRACT OR PHYSICAL) MODELS

- **Static Model:** is the one which describes relationships that do **not** change with respect to time.
 - An architectural model of a house which helps us visualize floor plans and space relationships is a **static physical model**.
 - An equation relating the lengths and weights on each side of a playground seesaw is a **static mathematical model**.
- **Dynamic Model:** is the one which describes time-varying relationships.
 - A wind tunnel which shows the aerodynamic characteristics of proposed aircraft designs is a **dynamic physical model**.
 - The equations of motion of the planets around the sun constitute a **dynamic mathematical model** of the solar system.

Classifications of Models

ANALYTICAL OR NUMERICAL (STATIC OR DYNAMIC) MATHEMATICAL MODELS

- **Analytical Model:** is the one which is solved by using the deductive reasoning of mathematical theory.
 - An M/M/1 queuing model, a Linear Programming model, a Mixed Integer Linear Programming model, a nonlinear optimization model are examples of analytical models.
- **Numerical Model:** is the one which is solved by applying computational procedures.
 - Finding the roots of a nonlinear algebraic equation, $f(x) = 0$, using the method of Interval Halving or Simple Iteration involves the use of a numerical model.
 - **System Simulation is considered to be a numerical computation technique.**

Classifications of Models

LINEAR OR NONLINEAR (STATIC OR DYNAMIC) MATHEMATICAL MODELS

- **Linear Model:** is the one which describes relationships in linear form.
 - The equation $y = 3x + 4z + 1$ is a linear model.
- **Nonlinear Model:** is the one which describes relationships in nonlinear form.
 - The equation $F = (2x + 4z - 2) / (3y - x)$ is a nonlinear model.

Classifications of Models

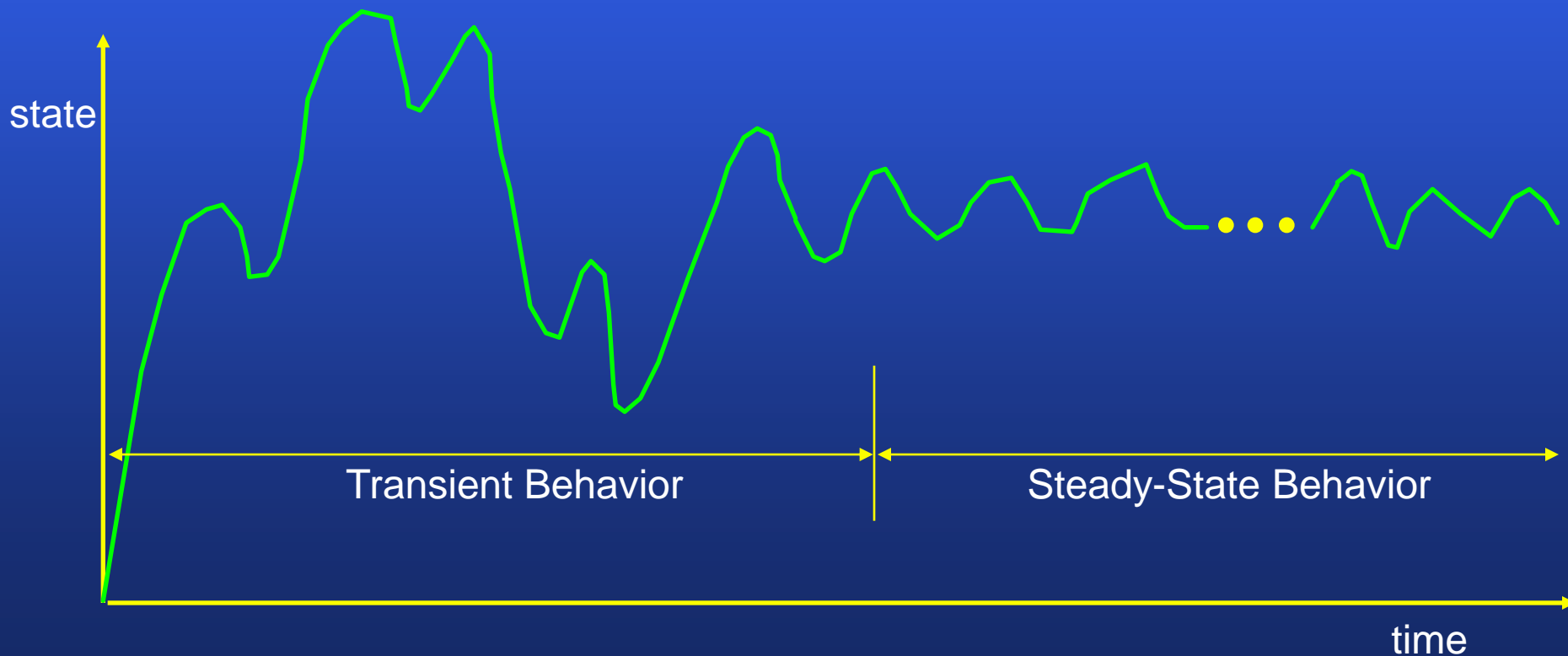
STABLE OR UNSTABLE MATHEMATICAL MODELS

- **Stable Model:** is the one which tends to return to its initial condition after being disturbed.
 - Like a simple pendulum that is set in motion, it may overshoot and oscillate, but the disturbances decline and die out.
- **Unstable Model:** is the one which may or may not come back to its initial condition after being disturbed.

Classifications of Models

STEADY-STATE OR TRANSIENT MATHEMATICAL MODELS

- **Steady-State Model:** is the one whose behavior in one time period is of the same nature as any other period.
- **Transient Model:** is the one whose behavior changes with respect to time.



Classifications of Models

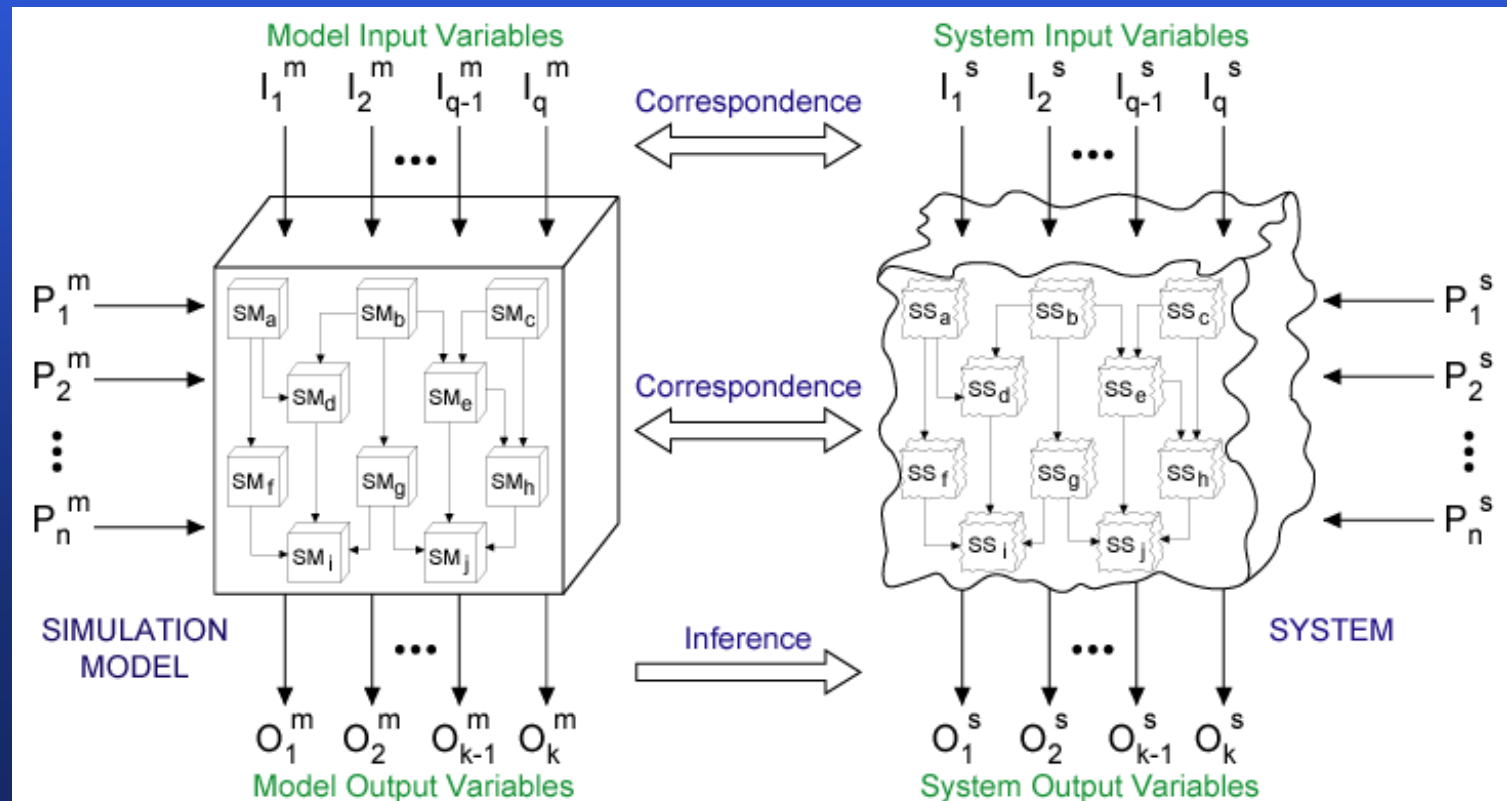
DESCRIPTIVE OR PRESCRIPTIVE (NORMATIVE) MODELS

- **Descriptive Model:** is a model which describes the behavior of a system without any value judgment on the “goodness” or “badness” of such behavior.
 - The equation $F \text{ (orce)} = M \text{ (ass)} \times A \text{ (cceleration)}$ is a descriptive model.
 - **All simulation models are descriptive models.**
- **Prescriptive or Normative Model:** is a model which describes the behavior of a system with a value judgment on the “goodness” or “badness” of such behavior.
 - A Linear Programming model, a Mixed Integer Linear Programming model and a nonlinear optimization model are examples of prescriptive models. When solved, these models provide a description of the solution as optimal, suboptimal, feasible, infeasible, etc.

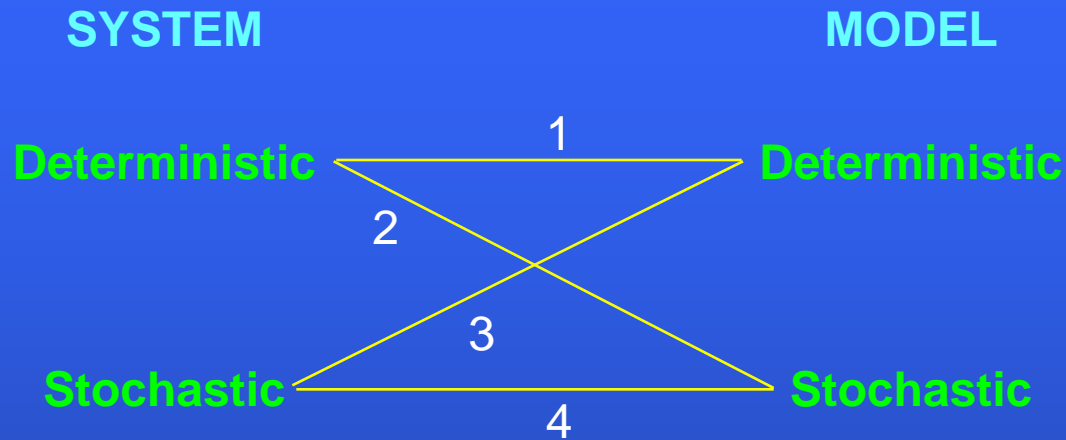
Classifications of Models

Two types of simulation exist with respect to the way the model is driven:

- **SELF-DRIVEN (Distribution-Driven or Probabilistic) SIMULATION** is the one which uses random numbers in sampling from probability distributions so as to drive the model.
- **TRACE-DRIVEN (or Retrospective) SIMULATION** is the one which uses the actual input data traced during the operation of the real system so as to drive the model.



Classifications of Models



1. The mathematical equations that constitute the model of the motion of a missile can be solved analytically to predict system behavior.
2. Evaluation of a definite integral by means of a stochastic process (**Monte Carlo Simulation**).
3. Generation of pseudo-random numbers on a computer.
4. Simulation is normally used in this case.

Classifications of Models

MODELS

```
graph TD; MODELS --> SIMPLE; MODELS --> COMPLEX;
```

SIMPLE

- Static
- Deterministic
- Linear
- Continuous
- Steady State
- Aggregate
- Stable

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COMPLEX

- Dynamic
- Stochastic
- Nonlinear
- Discrete
- Transient
- Detail
- Unstable